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DYNAMIC ANALYSIS OF DELAYED DAMPER SYSTEM IN ENGINEERING STRUCTURES

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The delayed damper (DD) is a new active vibration absorption technique that uses time-delayed partial state feedback to generate ideal resonance on a passive vibration absorber. It has many attractive features such as real-time adjustability, ease of implementation, and total suppression of vibration for tonal frequency disturbances. In this paper, a major advantage of engineering structures analysis is the reduction of characteristic roots from infinite to finite numbers and the consequent simplicity in the dynamic analysis of the controller. The dynamic model principle is employed to design controllers for the structure. The system is examined by simulation. It is shown that engineering structures control application for DD yields better vibration suppression considering the sampled control structure in implementations.

1. INTRODUCTION

Passive vibration absorbers have long been used as substructures that are attached to the primary structure to suppress the externally excited vibrations. When properly designed, these simple structures can be quite effective within the narrowband of frequencies, for which they are tuned. For the vibration created by tonal excitations, this ground rule of absorption dictates that the primary structure that is harmonically disturbed can be brought to rest if the vibration absorber attached to it has ideal resonance features at the frequency of concern. However, ideal resonance can be achieved only if the absorber has no damping, which is not practicable as every physical system has some degree of damping. Therefore, passive vibration absorbers cannot completely suppress the vibrations of the primary structure. In addition, if the disturbance frequency is not fixed (i.e., time varying), even a damping free vibration absorber is not desirable, since the attachment of a single-degree-of-freedom (s.d.o.f.) absorber to a primary system introduces new resonant frequencies, the suppressed resonant frequency of the primary structure alone. If the disturbance frequency is close to this new resonant frequencies, the response of the dynamic structure can be worse than that of the primary structure alone. For this reason, extensive research has been conducted for an optimum absorber that would minimize the primary response for a wideband of disturbance frequencies.

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A novel active vibration absorption technique, delayed damper (DD), has been introduced by Chen and Xu. As explained in this reference, the core idea is the use of a time-delayed position feedback for tuning the vibration absorber. When the proportionality gain and the feedback delay are properly selected, this simple control converts the absorber into a real-time tunable resonator at a desired frequency. Thus, the name delayed damper arose. When attached to a primary structure, the resonator removes all oscillations from the primary structure at the point of attachment, at its resonance frequency. The DD has many attractive features such as theoretically infinite frequency range of effectiveness, online adjustability, decoupled control (from the primary), and simple implementation. The DD control parameter selections and ensuing stability issues are studied in continuous domain. When the implementation is digital, an alternative way is to design the DD controller in discrete domain, which is the primary structure theme of this paper. It is shown that in a digital implementation, the DD tuning frequency range is no longer infinite; instead, it has an upper limit that depends on the sampling rate of the controller. Simulation results show that in a digital implementation, the performance of the DD is better if the controller is designed in the discrete domain.

Vibration is a significant issue in many structural engineering systems. The DD is a reliable passive control device but has significant limitations in structural applications where disturbances are wideband, it is effective only over a single narrowband of frequencies. The DD shows promise, but additional research is required to ascertain its utility for control in engineering structures. The objective of this paper, then, is to develop and analyze the DD for vibration suppression. This will be done by designing and analyzing the controller, first for a s.d.o.f. system and then for a structure. The efficacy of the DD control strategy will be verified experimentally for the structure.

The layout of this paper is as follows: in section 2, the DD system is introduced. A systematic way is presented in section 3 for analyzing the stability of the dynamic system. The application to the control of the structure is given in section 4. Simulation results are presented in section 5.

2. THE DELAYED DAMPER SYSTEM

The system model is shown in Figure 1,which consists of the mass, spring, viscous damper, and a feedback control force $u(t)$. The equations of motion for this system are given by

$$\begin{bmatrix} m & 0 \\ 0 & m_a \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{x}_a \end{bmatrix} + \begin{bmatrix} c + c_a & -c_a \\ -c & c_a \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{x}_a \end{bmatrix} + \begin{bmatrix} k + k_a & -(k_a + g(t-\tau)) \\ -k & k_a + g(t-\tau) \end{bmatrix}\begin{bmatrix} x \\ x_a \end{bmatrix} = \begin{bmatrix} F_0 \sin \omega t \\ 0 \end{bmatrix}, \tag{1}$$

where X_a is the displacement of the absorber mass and X is the displacement of the primary mass. Equation (1) roots may be found by Laplace domain

$$X = \frac{(k_a - m_a\omega^2) + \mathrm{j}c_a\omega + g\mathrm{e}^{-\mathrm{j}\tau\omega}}{EM} F_0, \tag{2}$$

$$X_a = \frac{(k + \mathrm{j}c\omega)}{EM} F_0. \tag{3}$$

Here,

$$EM = \{[(k - m\omega^2) + \mathrm{j}c\omega] + (k_a + \mathrm{j}c_a\omega)\}\{[(k_a - m_a\omega^2) + \mathrm{j}c_a\omega] + g\mathrm{e}^{-\mathrm{j}\tau\omega}\} - (k_a + \mathrm{j}c_a\omega)[(k_a + \mathrm{j}c_a\omega) + g\mathrm{e}^{-\mathrm{j}\tau\omega}], \tag{4}$$

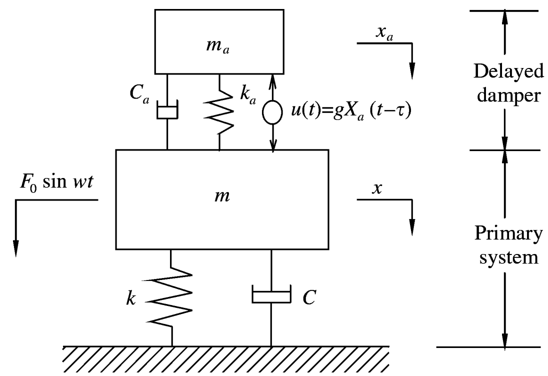

Figure 1. Model of the dynamic system.

where ω is the tuning frequency, g is the feedback gain and τ is the feedback delay.

Using the relationships

$$\omega = \omega_c, \qquad g = g_c = \pm\sqrt{(c_a\omega_c)^2 + (k_a - m_a\omega_c^2)^2}, \tag{5, 6}$$

$$\tau = \tau_c = \frac{1}{\omega_c}\left\{\tan^{-1}\left[\frac{c_a\omega_c}{m_a\omega_c^2 - k_a}\right] + 2(l+1),\right\} \quad l = 1, 2, \ldots, \tag{7}$$

where ω_c is the DD crossing frequency, g_c is the crossing gain for the DD and τ_c is the crossing delay for the DD; equations (2) and (3) can be given by

$$x = 0, \quad x_a = -\frac{F_a}{m_a\omega_c^2}. \tag{8}$$

In this case, Roots (8) yields simple analytical viewpoint is, varying the control parameters g_c and τ_c, one can change the amplitude and the resonance frequency of the DD in real time.

Now that a passive absorber is converted into a perfect resonator at the desired ω_c (resonance frequency), it can be deployed as an active vibration absorber as depicted in Figure 1. It is easy to show that if the excitation force were a simple harmonic function such as $F_0 \sin(\omega_c\, t)$, then the DD would remove the entire oscillation energy delivered to the primary structure at frequency ω_c. Consequently, the primary structure would remain at rest, while the DD creates negating forces jointly with the restoring elements (spring + damper). Note that the relative or absolute nature of the feedback (i.e., X_a) is irrelevant for this absorption. The only difference is in the dynamic system behavior, which is treated next. Another key point is that the frequency of excitation, ω_c, can be simply detected from the time trace of X_a. In many reported experimental works, authors utilized zero crossing observations for detecting this feature.

3. STABILITY OF THE DYNAMIC SYSTEM

The DD is a stand-alone device, that is, its control is decoupled from the primary structure. For a total vibration absorption at a given frequency, the DD is kept at marginal stability. The simple control used for this purpose, automatically sets the stability nature of the dynamic system. The only determining factor in this process is the tuning frequency of the DD. Therefore, prior knowledge of an operating frequency range is needed, within which not only the DD stability is guaranteed but also that of the system. We present this issue next.

In general, the state-space representation of the dynamics for an nd.o.f. primary with a s.d.o.f. DD can be written as

$$x(t) = Fx(t) + Gu(t), \tag{9}$$

where $x = [x_a, x_1, x_2, \dots, x_n, x_a, x_1, x_2, \dots, x_n]^{\mathrm{T}} \in R^{2(n+1)}$ is the state vector, $F \in R^{2(n+1)\times 2(n+1)}$, $G \in R^{2(n+1)}$ are constant matrices and $u \in R$ is the control. In discrete domain, the dynamics of equation (9) take the form

$$x(k+1) = \Psi x(k) + \Gamma u(k). \tag{10}$$

where Ψ and Γ are of appropriate dimensions. Considering the control, an augmented state model can be obtained as

$$\theta(k+1) = \Sigma\theta(k), \tag{11}$$

where

$$\Sigma = \begin{bmatrix} \Psi & \Gamma_1 & \Gamma_2 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & & & & & \\ 0 & & & 1 & & & & \\ \cdot & & & & \cdot & & & \\ \cdot & & & & & \cdot & & \\ \cdot & & & & & & \cdot & \\ 0 & & & & & & & 0 \\ I & & & & & & & 0 \end{bmatrix},$$

$$\Gamma_1 = \eta\Gamma, \quad \Gamma_2 = (1-\eta)\Gamma, \quad I = [1, 0, \dots, 0] \in \Re^{2(n+1)}$$

and

$$\theta = [x, x_a(k-N-1), x_a(k-N), \dots, x_a(k-1)^{\mathrm{T}} \in R^{2n+N+3}.$$

For the stability of the dynamic system, the eigenvalues of Σ should lie within the unit circle. This can be easily checked (because of the finite number of roots involved) once the control parameters are calculated for a given frequency ω_c. Repeating this operation for varying ω_c values, the range of frequencies is determined for which the given DD + primary structure is stable. Another line of research is followed presently on the determination of the robustness features of the DD implementation. This effort yields an adaptively robust control, which compensates against the structural variations within the absorber section.

4. APPLICATION TO CONTROL OF THE STRUCTURE

To ascertain its effectiveness the DD control was investigated through a series of simulations and experiments applied to a structure with the properties given in Table 1.

The configuration of the tested system is shown in Figure 2. A shaker was positioned 260 mm from the fixed support, and the accelerometer and actuator were effectively located at the structure tip. It was necessary to determine the transfer function from the actuator input to the accelerometer output in order to develop the controller. A white-noise input

TABLE 1

Specifications of the structure

Material	Cross-section	Length
Alloy steel	33 mm × 23 mm	600 mm

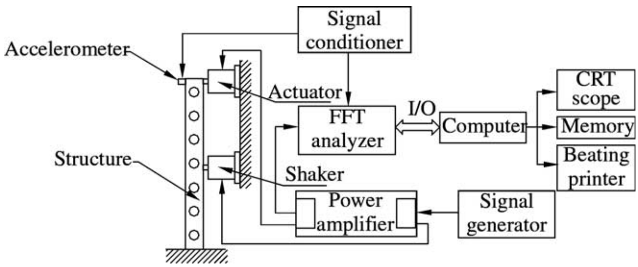

Figure 2. Schematic structure and control system.

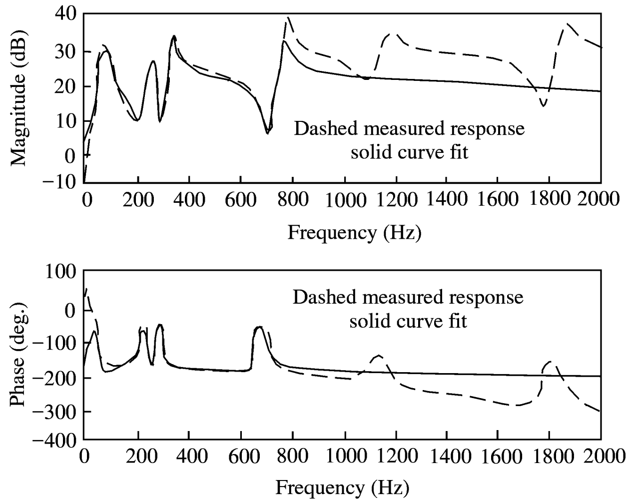

Figure 3. Comparison of the 16th order curve fit model and the measured frequency response.

voltage was applied to the actuator, and the frequency response of the structure was measured using a HP35670A Fourier analyzer. The coherence function was found to be nearly lower for most of the frequency range examined, indicating a fairly accurate measurement.

The experimental frequency response function was curve fitted with a 16th order transfer function using MATLAB. The curve fit sampling frequency was 5120 Hz, which corresponds to the sampling frequency of the Fourier analyzer. The frequency range of primary concern was 100–700 Hz; thus, only three bending modes of the structure were included in this transfer function, as seen in Figure 3.

Table 2

Comparison of open-loop and theoretically best closed-loop responses

Disturbance frequency (Hz)	Open-loop root mean square accelerometer output	Closed-loop root mean square accelerometer output	Response reduction (%)
80	0·9033	0·0071	99·21
160	0·4141	0·0098	97·63
240	1·7195	0·0688	96·00
320	3·9646	0·1813	95·43
400	1·4206	0·0368	97·41
480	1·0772	0·0245	97·73
560	0·8414	0·0072	99·14
640	0·4626	0·0015	99·68
720	1·3892	0·0124	99·11
800	3·5793	0·0231	99·35
880	2·1162	0·0236	98·88
960	1·6687	0·0367	97·80

Table 3

Comparison of open- and closed-loop responses with the controller containing the band- and high-pass filter

Disturbance frequency (Hz)	Open-loop root mean square accelerometer output	Closed-loop root mean square accelerometer output	Response reduction (%)
240	1·7195	1·4771	14·10
320	3·9646	1·8195	54·11
400	1·4206	0·0915	93·56
480	1·0772	0·0332	96·92
560	0·8414	0·1937	76·98

5. SIMULATION

The controlled structure was simulated using the Control System Toolbox in MATLAB. First, the maximum possible reduction in response magnitude was determined for the system. A sampling frequency of 5120 Hz selected, and defined previously was set to 64,resulting in an 80 Hz spacing between suppressed frequencies. The theoretical 1imit on the reduction is reached with the exact inverse of the plant and a low-pass filter with a very high cut-off frequency. The root mean square values of the accelerometer output are compared in Table 2 for the open-and closed-loop systems at the attenuated frequencies.

The average reduction in response magnitude at the cancelled frequencies was 98% over the range of 80–960 Hz. Slight improvements would be possible if a higher order were used; then the filter would not be needed, and the response magnitude at integer multiples of 80 Hz would theoretically be zero. Obviously, a perfect model is not obtainable, so this level of performance cannot be reached; however, it is possible to achieve similar results on much smaller frequency intervals when a more realistic controller is utilized.

Finally, the exact controller used in the experimental control system was simulated. This transfer function contained both the band-pass filter and the exact inverse of the plant.

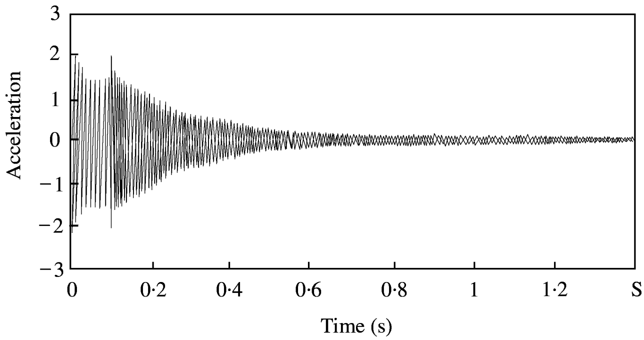

Figure 4. Time history of the simulated controlled structure system to a 490 Hz disturbance force.

Precisely at the suppressed frequencies in the pass band, the controller achieved almost the same performance as in the above simulations. The reductions in the root mean square (r.m.s.) acceleration at 400 and 480 Hz were greater than 90%, as shown by the response magnitudes summarized in Table 3. It should be noted that the notches in the frequency response are much narrower than those of the other simulated systems. In addition, disturbances at frequencies only slightly lower than multiples of 80 Hz are magnified within the pass band. This is caused by a closed-loop pole in close proximity to the suppressing zero. These peaks can be decreased, but performance at the attenuated frequencies will suffer. A time-domain simulation is presented in Figure 4, where the structure was excited at 480 Hz, and the controller was turned on at time $t = 0{\cdot}1$ s.

6. EXPERIMENTAL RESULTS

The controller was implemented on a digital signal processor (DSP) board using, primarily, the C programming language. The repetitive component was implemented in assembly language and a circular buffer structure was used. Both the high-pass and the band-pass were applied. All of the data were fed directly into MATLAB, and the control gains could also be set while the controller was operating within the MATLAB environment. The best performance, as predicted by the simulations, occurred at disturbance frequencies of 400 and 480 Hz. Table 4 summarizes the results at various disturbance frequencies. The maximum achievable performance for this controller gave approximately an 80% reduction in vibration response magnitude. The other controllers that were simulated above were implemented on the DSP but without success as the structure system became unstable when the control force was applied. As might be expected, the experimental implementation did not achieve the level of performance predicted by the simulations, probably because of imperfect modelling and experimental signal and measurement noise.

7. CONCLUSIONS

In this paper, the DD has successfully been applied for active vibration suppression. Through the simulation, the DD concept has been shown to be an effective strategy for suppressing periodic disturbances at known frequencies. Nevertheless, stability, becomes an issue due to the large feedback gain at high frequencies. Key to the successful

TABLE 4

Summary of experimental results comparing open- and closed-loop responses

Disturbance frequency (Hz)	Open-loop root mean square accelerometer output	Closed-loop root mean square accelerometer output	Response reduction (%)
320	1·5787	1·5567	1·39
400	0·6586	0·1623	76·08
480	0·7007	0·1429	79·61
560	0·8947	0·6483	27·54

implementation of the DD was the development of stability criteria. A simple design guideline for the low-pass filter within the controller that related the magnitude of the filter to the unmodelled dynamics was applied, thus guaranteeing stability. The DD scheme employing the dynamic model principle has many advantages over other controllers designed to suppress narrowband disturbances. First, the majority of other systems can attenuate only one or two disturbance frequencies, while the number of frequencies the DD can suppress is limited only by hardware constraints, stability criteria, and modelling issues.

Simulations of the controlled structure presented here show that the DD system scheme is effective at rejecting disturbances occurring at a reference frequency and its integer multiples. More than a 90% reduction in vibration response magnitude is possible when the DD is configured properly. A systematic way for analyzing the stability of DD is presented, which yields a table of stable operating frequencies as an analysis tool. It has been shown that the DD system is of great theoretical and practical significance by the dynamic analysis.

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